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Fair Sharing of Backup Power Supply in Multi-Operator Wireless Cellular Towers

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***Abstract*—Keeping the continuity of wireless base stations operation and providing uninterrupted communications services can save billions of dollars as well as human lives during natural disasters and/or electricity outages. Towards this end, wireless operators need to install backup power supplies whose capacity is sufficient to support their peak power demand, thus incurring a significant capital expense. Hence, pooling together the backup power supplies and sharing it among the co-located wireless operators can effectively reduce the capital expense, as the backup power capacity can be sized based on the aggregate demand of co-located operators instead of individual demand. Turning this vision into reality, however, faces a new challenge: how to fairly share the backup power supply? In this paper, we propose fair sharing of backup power supply by multiple wireless operators based on the Nash Bargaining Solution (NBS). Furthermore, we extend our analysis to multiple time slots for emergency cases in which the study the backup energy sharing based on model predictive control and NBS subject to an energy capacity constraint regarding future service availability. Our simulations demonstrate that the sharing backup power/energy improves the communications service quality with lower cost and consumes less base station power than the non-sharing approach.**

***Index Terms*—Backup Power, Base Station, Fair sharing, Decentralized Optimization.**

I. INTRODUCTION

Nowadays, more and more services are heavily dependent on mobile communication and data services. Therefore, service availability becomes an essential requirement for every mobile network operator. However, cellular base stations (BSs) are mostly operated on utility electricity that is cost effective but unstable, especially during natural disasters. Due to the aging power infrastructure, extreme weather and natural disasters, power outages have become increasing more frequently, making it extremely challenging to keep communications service continuity. For example, communications service interruptions affected by power outages are a daily norm in many developing countries [1], while even in developed economies such as the United States, communications service outages are also proliferating [2].

To ensure communications service continuity, wireless operators have commonly installed backup power supplies alongside their BSs. The necessity of improving service availability during power outages has drawn significant attention. The Federal Communications Commission (FCC) has proposed a mandate that carriers must increase or provide sufficient emergency/backup power at their cell sites [3]. Specifically, after the severe devastating earthquake happening in 2011, the Japanese KDDI Corporation had recommended increasing deployment of emergency power generators as an effort to attain early recovery of coverage in the event of large-scale disaster [4]. During power emergencies, backup power supply at the BSs is required to ensure normal operation of the cellular BSs without interruption.

There are multiple options to supply backup power to BSs during power outages, such as diesel generator, lead-acid battery, li-ion battery and fuel cell. Currently, diesel generator is widely set up for many systems; however, it has some drawbacks, such as pollution, high noise, and heavy weight, which is not suitable for many urban wireless tower installations. Further, diesel generators cannot instantly start in the event of a power outage, and hence service interruptions cannot be fully avoided with diesel generators alone. In addition, battery is also a common option for backup power, but its high capital investment and maintenance cost have made it less and less appealing. More recently, fuel cells have been extensively studied in an attempt to improve technical performance, reliability and reduce environmental issues. For a typical site with one BS, the CO_2 emission is reduced 34.4 tons per years using a hydrogen fuel cell compared to gas and diesel generator [5]. Moreover, fuel cell is highly robust in rugged areas and has a much lower maintenance cost than batteries. For these advantages, fuel cell is emerging as one of the most popular options for many applications including BS backup power.

While multiple backup power options are available, they are all very expensive. Especially for the increasingly more popular fuel cell, one kW of fuel cell capacity costs 2500-3000 dollars, making it a billion dollar project for any major wireless operators to install backup power for its BSs [5]. This

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creates an impediment to implementing the FCC regulation regarding backup power at the cellular BSs. Although FCC recommends the installation of at least 8 hours backup power installation, many BSs do not have enough backup power, even for major wireless carriers, due to the high capital cost.

In view of the high capital cost, sharing the precious backup power resource emerges as a key opportunity to lower the cost and benefit all participating wireless operators. Indeed, backup power sharing can be easily implemented with almost no changes to the co-located sites, where many wireless operators already shared the tower infrastructure and physically co-locate their BSs. In the report [6], tower sharing allows operators to cut CapEx, e.g., infrastructure cost for operators is reduced by 16% to 20%. Tower sharing also becomes the most common form of passive infrastructure sharing with infracos (i.e., the shared capital telecom infrastructure companies) in Europe, Japan, and many other regions [6]. In addition, independent tower companies have become prevailing in India, China, and Southeast Asia since 2015, and has been growing rapidly in Sub-Saharan Africa [6]. Since most mobile towers are owned and leased by the third party mobile tower companies, co-located BSs have a strong presence in the United States [7]. More importantly, in an attempt to decrease the deployment cost of stationary standby generators or fuel cells and to avoid the logistic problems with parking and fuel delivery access for portable generators, some task forces of FCC have begun to study and recommend the sharing of power supplies [8]. Consequently, on top of tower sharing, backup power sharing among multiple wireless operators can be easily deployed and viewed as an integral element of infrastructure sharing [7]. When the backup power is owned by the tower operator and shared among multiple co-located wireless operators, it is typically sold to these operators in advance with fixed amounts by contract without accounting for fairness or runtime workload conditions. Due to the limited and insufficient energy capacity, a fair sharing mechanism can help to improve the delay performance and save more backup energy.

Despite the economic advantage and benefit, a major issue is how to fairly share the backup power among multiple participating operators and making them better off. In this paper, we study the under-explored problem - *fairness of backup power sharing in multi-operator cellular towers where wireless operators can associate their own traffic loads (i.e., route their power demand) to different towers in a fair manner*. Towards this end, we adopt Nash Bargaining theory, which is designed for a cooperative game that helps participants achieve fairness and Pareto optimal solutions [9]–[12]. Intuitively, operators can make an agreement to maintain the service by using shared backup power in a collaborative manner if they attain greater utility than non-cooperating. Our proposed cooperative strategy helps operators improve the network performance compared to no sharing approach in which the operators use their own individual backup power alone.

In addition to the power capacity constraint shared by multiple operators, we also consider energy capacity constraints during extended grid outages. In such situations, BSs can only use the backup power to prolong the communication services while the energy of power supplies are constantly depleted over usage time. Under energy limitation, sharing backup power supplies among the operators should also consider of future service availability, which requires a deliberate planning approach. Hence, Model Predictive Control (MPC) is applied to design Nash Bargaining Solution (NBS) for the planning problem with guaranteed fairness.

Recently, there is an increasing interest in sharing power studies for mobile networks. *First*, in the state-of-the-art on sharing renewable power [11], [13], [14], the authors propose hybrid power models, in which BSs can receive power from both electric grid and renewable energy. Even though BS power demand exceeds the renewable energy capacity and battery storage, operators can receive additional power from the grid. On the other hand, we explores the uninvestigated sharing backup power problem in which the available backup power is a hard constraint for each operator during an emergency grid outage. *Second*, the works [11], [13] focus on sharing power among BSs without multi-operator consideration. In the recent work [14], the authors consider sharing power storage among the multi-operator at a single site while in [15], [16], the authors propose the multi-operator cooperation based on roaming/offloading traffic loads and low-utilized BSs switching-off operation for multiple sites. Nevertheless, our proposed model exploits multi-operator backup power sharing and user association decision among multiple sites in a considered region. Accordingly, power demand of BSs at the co-located multi-operator sites can be regulated by routing user traffic loads among these sites. In this scenario, the amount of received backup power is the only power source of the system which needs to be shared among all operators. Individually, each operator makes a decision on user association such that they can balance the BSs load and optimize the average delay performance based on the flow-level analysis. Different from the existing works on energy sharing, we adopt a different utility function (i.e., flow-level delay cost). *Third*, under the limitation of the available backup power, a cooperative approach using Nash Bargaining Solution can guarantee the fairness of operators' gain in terms of delay performance when participating in the cooperative solution. Therefore, the integration of user association problem and the sharing limited backup power are analyzed in our work. Specifically, Table I summarizes the comparison between our work and the other BS energy sharing papers. In this table, RE stands for Renewable Energy and SG stands for Smart Grid.

The most relevant work from BSs energy management collaboration to ours has been proposed by Xu et al. [15], which treats the renewable energy as some "good-to-have" resources. Accordingly, operators can receive additional power from the grid when the BS power demand exceeds the battery capacity and RE generation. However, in our work, backup power usage is restricted by a hard constraint for each operator during an emergency grid outage. We exploit the co-located wireless operators at multiple sites model, in which power demand of BSs at the co-located sites can be regulated by routing user traffic loads among these sites, while the work [15] leverages BSs sleep, energy group buying and wireless load sharing for saving energy cost. Besides, our cooperative

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Related paper	*Farooq et al. 2017* [13]	*Xu et al. 2016* [11]	*Xu et al. 2016* [15]	*Bousia et al. 2016* [16]	*Maria et al. 2017* [14]	*Our work*
Power sources	RE, SG	RE, SG	RE, SG	SG	RE, grid (not SG)	Backup energy
System model	BSs	BSs	Multi-operator at multiple sites	Multi-operator at multiple sites	Multi-operator at single site	Multi-operator at multiple sites
Design	Physical energy sharing links among BSs, and virtual links via SG	Energy group buying, downlink CoMP cooperation	BSs sleep, roaming/offloading operation	BSs sleep, roaming/offloading operation	Renewable, battery storage sharing	User association among sites, backup power cooperation
Solution method	Hierarchical clustering	Convex optimization	Repeated Nash bargaining	Non-cooperative game	Coalition game, Shapley value	Nash Bargaining Solution, JP-ADMM
Optimization objective	Total energy cost	Total energy cost	Individual energy cost with fairness	Network cost (energy, roaming)	Only fairness in energy sharing	Flow-level cost with fairness

TABLE I: Related work comparison.

sharing approach is designed based on Nash Bargaining solution and model predictive control for planning during the required period while the paper [15] proposes a repeated Nash Bargaining scheme. Our proposed decentralized algorithms are considered as the bargaining processes to achieve Nash Bargaining solution without revealing traffic information for the backup power fair sharing. In summary, the key novelty of our study is that we propose the fair backup power sharing among wireless operators as a cost-effective approach to improve the communications service quality. Concretely, we make the following contributions:

- In Section II, we apply the analytical framework of flow-level delay-optimal user association [17] among cellular BSs of a single operator into the co-located multi-operator sites in wireless networks. Accordingly, the objective of operators is minimizing their delay performance in terms of the flow-level cost and load balancing among BSs.
- In Section III, we develop a scheme to fairly share the backup power supply among the operators by applying the NBS. For practical implementation, we design a decentralized algorithm based on Jacobi-Proximal Alternating Direction Method of Multipliers (JP-ADMM) approach with limited information exchange among the operators to solve the bargaining problem. Then, we provide numerical studies based on practical settings of cellular BSs to demonstrate the effectiveness of the proposed backup power sharing scheme. Our sharing backup power approach can reduce the flow-level cost in terms of delay and improve power efficiency compared to no sharing strategy.
- In Section IV, we consider the bargaining problem with regard to the energy capacity limitation of the backup power. We apply NBS with MPC to make planning decisions for future service availability consideration, and design a decentralized algorithm to solve the *backup energy fair sharing and planning* problem. Numerical studies for an planning shows the lower flow-level cost and higher remaining energy of sharing mechanism than individual MPC power planning implementation.

TABLE II: **Notation**

Var	Definition
i	Index denoting cellular operator
j	Index denoting BS site
$\lambda(x)$	Arrival rate per unit area at location x
$\mu(x)$	Mean data size at location x
$\gamma(x)$	Traffic load density at location x
$c_{ij}(x)$	Transmission rate of user located at x and served by BS j of operator i
$\beta_{ij}(x)$	System load density at location x or BS j of operator i
$p_{ij}(x)$	Routing probability associated with location x and BS j of operator i
ρ_{ij}	Load utilization of BS j of operator i
$\phi(\rho_i)$	Flow-level cost of operator i
$\psi_{ij}(\rho_{ij})$	Power consumption of BS j of operator i
Q_{ij}	Maximum operational power of BS j of operator i
B_{ij}	Maximum power of backup power supply of BS j of operator i
$\hat{\phi}_{ij}$	Disagreement point of NBS associated with operator i
L_i	Expected total number of flows of operator i

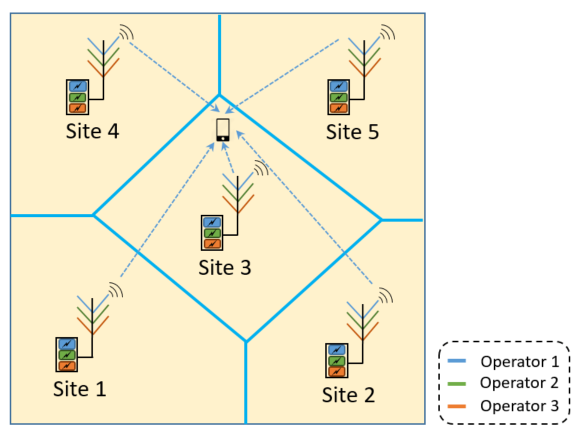

Fig. 1: User association at multi-operator sites model.

II. System Model

We apply the infrastructure-based wireless network model from multiple BSs of a single operator [17] to the co-located multi-operator sites as shown in Figure 1. Mobile users in a considered region $\mathcal{L} \in \mathbb{R}^2$ are served by a set $\mathcal{G}$ of operators. Rather than the individual deployment, each operator i has a BS set $\mathcal{H}_i$, which is located at different sites. At each site j, operators share cellular BS infrastructure and have backup power supplies. Figure 1 illustrates that a mobile terminal (MT) of operator 1 can associate with one of the co-located multi-operator sites. Even though being closest to the BS at site 3, this MT may be associated with farther BS at other sites through mobile hand-off if the BS at site 3 is heavily utilized. Since our work analyzes the sharing power of operators at co-located sites, we focus on the downlink scenario, in which BS power consumption is linearly increasing with the mobile traffic load.

At any location $x \in \mathcal{L}$, the traffic flows follow an inhomogeneous Poisson point process with arrival rate per unit area $\lambda(x)$. For simplicity, the arrival traffics can be modeled as user flows (i.e., data requests) with random sizes following independent distribution with mean $\frac{1}{\mu(x)}$. Then, the *traffic load density* at the location x is defined as $\gamma(x) = \frac{\lambda(x)}{\mu(x)}$ in [17]. We assume $\gamma(x) < \infty$ for all $x \in \mathcal{L}$. The spatial traffic variability is captured in the traffic load density expression.

Following the literature [17], [18], we consider the path-loss model to capture the average channel quality between user locations and BSs. In addition, instead of dynamic inter-cell interference, we only consider the static Gaussian-like noise inter-cell interference with interference randomization or fractional frequency reuse [19]. The fractional frequency reuse provides a strategy to mitigate interference and make interfered cells sufficiently separated if they operate on the same frequency. At location x, the transmission rate served by BS j of operator i is denoted by $c_{ij}(x)$ which follows Shannon capacity

$$c_{ij}(x) = BW \cdot \log_2 \left(1 + \frac{P_{ij} g_{ij}(x)}{\sigma^2 + I_{ij}(x)}\right) \tag{1}$$

where P_{ij} denotes the transmission power of the operator i at BS j and $g_{ij}(x)$ denotes the channel gain from the BS j of operator i to the MT at location x, including path loss, shadowing, and other factors. In addition, σ^2 denotes noise power and $I_{ij}(x)$ denotes the average interference seen by the MT at location x. Various available radio propagation models can be used to predict the path loss in dB and account for shadow fading effect. As a result, transmission rate becomes location dependent.

The *system-load density* [17] is denoted by $\beta_{ij}(x) = \frac{\gamma_i(x)}{c_{ij}(x)}$, which defines the fraction of active transmission time required to deliver the traffic load $\gamma_i(x)$ of operator i from BS j to location x. The user associated routing probability vector for each operator i is denoted by $\mathbf{p}_i(x) = \{p_{ij}(x)\}$ for all $x \in \mathcal{L}$ and $j \in \mathcal{H}_i$.

Definition 1 (Feasibility): The set $\mathcal{F}_i$ of feasible BS loads (or utilization) of the operator i, i.e., $\rho_i = \{\rho_{ij}\}$ for all $j \in \mathcal{H}_i$ is defined as follows

$$\begin{aligned} \mathcal{F}_i = \big\{\rho_i \,|\rho_{ij} = \int_{\mathcal{L}} \beta_{ij}(x) p_{ij}(x) dx \\ 0 \le \rho_{ij} \le 1 - \epsilon, \sum_{j \in \mathcal{H}_i} p_{ij}(x) = 1, \\ 0 \le p_{ij}(x) \le 1, \forall j \in \mathcal{H}_i, \forall x \in \mathcal{L}\big\}, \end{aligned}$$

where ϵ is an arbitrarily small positive constant. The sum of the routing probability of a traffic flow at any location x to all the BSs should be 1. The feasible set $\mathcal{F}_i$ was proved to be convex in [17].

A. Flow-Level Cost Model

In this work, we adopt the flow-level dynamic systems [17], which consider data requests (i.e., flows and file transfers) that are initiated randomly and leave the system after being served. This will capture the network performance as the stability of a queueing system. The user association problem from the dynamic flow-level model can be seen as a routing problem. By using this model, the load balancing issue of BSs is proposed in [17], in which MTs can be associated with farther low-utilized BSs in order to achieve better system performance in terms of average queueing delay of the operator. Furthermore, the stochastic traffic loads are modeled as inhomogeneous spatial distributions, enabling realistic traffic characteristic for system-level analysis of mobile operators. Based on the queueing analysis [17] for the M/GI/1 multi-class processor sharing system, the expected total number of flows of the operator i is calculated by $L_i = \sum_{j \in \mathcal{H}_i} \frac{\rho_{ij}}{1-\rho_{ij}}$. Since minimizing the expected total number of flows is equivalent to minimize the average delay according to Little Law, the average delay of a typical flow D_i of the operator i is as follows

$$D_i = \frac{L_i}{\Lambda_i} = \frac{1}{\int_{x \in \mathcal{L}} \lambda_i(x) dx} \times \sum_{j \in \mathcal{H}_i} \frac{\rho_{ij}}{1 - \rho_{ij}}. \tag{2}$$

For analytical purpose, we use the cost function for flow-level performance in [17] as follows

$$\phi(\rho_i) = \sum_{j \in \mathcal{H}_i} L_i + 1 = \sum_{j \in \mathcal{H}_i} \frac{1}{1 - \rho_{ij}}. \tag{3}$$

Minimizing the cost function $\phi(\rho_i)$ is equivalent to minimizing L_i, thus minimizing the average flow delay, which helps to improve user QoS of the operator i.

B. Base Station Power and Energy

According to [18], the BS power consumption increases with the increasing BS utilization and there are two kinds of power consumptions: fixed power consumption and the power consumption that are proportional to BSs utilization. Thus, the total power consumption of a BS is given by

$$\psi_{ij}(\rho_{ij}) = (1 - m_{ij})\rho_{ij} Q_{ij} + m_{ij} Q_{ij}. \tag{4}$$

where $m_{ij} \in [0, 1]$ is a portion of the fixed power consumption of the BS and Q_{ij} is the maximum BS's operational power when it is fully utilized, i.e., $\rho_{ij} = 1$, which includes power consumptions of transmit antennas, power amplifiers, and others. When $m_{ij} = 0$, BSs would ideally consume no power when idle, and gradually consume more power as the utilization increases.

Suppose that service time is divided into multiple time periods, each with an equal length Δ_t, the energy consumption of the BS in a time period is measured by the multiplication of power consumption and the amount of usage time Δ_t. The energy capacity of the backup power supply of BS j of operator i is denoted by C_{ij}, then the remaining available energy of the BS backup power for the next time period is defined by

$$E_{ij}(t+1) = E_{ij}(t) - \psi_{ij}(\rho_{ij}(t)) \times \Delta_t, \text{ where } E_{ij}(1) = C_{ij}.$$

In this paper, the operator performance is evaluated in terms of flow-level performance (3), i.e., the average delay depending on BSs utilization by queueing analysis. The utilization of these BSs determines the power usage according to (4). Therefore, when using backup power to prolong communication services (e.g., due to power outages), downsizing the maximum operational power for the economic purpose will negatively affect the BS performance. Specifically, for the user association problem with the backup power capacity of BS, some MTs cannot associate with a nearby BS with high traffic load density. Accordingly, these MTs are associated with more distant BSs with lower traffic load density. Due to the traffic density heterogeneity in different locations, the operators can have benefits of sharing the backup power to improve the delay performance. Conceivably, sharing backup power among the operators can not only reduce the capital costs but also improve the operator performance. Furthermore, in the emergency cases (e.g., power outages), the average delay minimization becomes the vital objective of operators which can also help to reduce backup power usage based on the lower BS utilization. Therefore, the fairness design based on average delay performance is our central analysis and we neglect the different service policies of operators. In addition, the differences among operators are *traffic rates, user location distribution,* and *spatial traffic density*.

In the next sections, we aim to design an effective backup power sharing mechanism for the participating operators co-located at the multi-operator sites so that every operator can benefit from sharing backup power supply among themselves. We study two scenarios: backup power supply sharing without energy capacity constraint in Section III and with energy capacity constraint consideration in Section IV.

III. Backup Power Sharing

Backup power sharing without energy capacity limitation consideration is useful for short-term power disruption. To enable the cooperation between operators for backup power sharing, we formulate a fair sharing problem based on the Nash Bargaining game [9], which can reduce the average flow delay better than no sharing scheme. We also design a decentralized algorithm to achieve the NBS.

A. Problem Formulation

We model the interaction between the operators at co-located sites as shown in Figure 1. Each operator minimizes its flow-level cost function (3), which is convex with respective to the BS loads. Independently optimizing user association with their own backup power would be trivial if the operators had no cooperation. In this case, the utility of operator i, denoted by $\hat{\phi}_i$, is determined by solving the following problem

No Backup Power Sharing (NBPS):

$$\begin{aligned} \min_{\mathbf{p}_i} \quad & \phi(\rho_i) && && (5)\\ \text{s.t.} \quad & \psi_{ij}(\rho_{ij}) \le B_{ij}, && \forall j \in \mathcal{H}_i, \\ & \rho_{ij} \in \mathcal{F}_i, && \forall j \in \mathcal{H}_i. \end{aligned}$$

The optimal user association of the **NBPS** problem represents the probability vector. Based on this probability vector, MTs should be associated with their corresponding BSs to minimize the flow-level cost faced by operator i at every site under the limitation of the BS's maximum operational power. Therefore, downsizing the maximum operational power decreases the number of MTs that can be associated with their closest BSs. Due to the limitation of backup power, the utilization of BSs located at larger density area will be higher, which forces more MTs to associate with farther BSs, thus lower transmission rate.

The question then arises: Is there any way that the operators can cooperate on sharing backup power to improve their performance, i.e., achieve $\phi_i \le \hat{\phi}_i, \ \forall i \in \mathcal{G}$ and such that:
a) The gains from cooperation are fair at a Pareto-efficient outcome?
b) Operators do not have to reveal any private information about their traffic loads?

To deal with the first question, we will resort to the NBS in the next paragraphs. For the second question, we also design a decentralized algorithm so that operators can protect their traffic load privacy in the next subsection.

1) Backup Power Fair Sharing using NBS: When the Nash Bargaining game is applied for the backup power fair sharing, the produced NBS of this cooperative game guarantees an outcome, which is not only Pareto-efficient but also proportional-fair [9], [20]. If the Nash Bargaining game cannot produce better delay performance for all operators, their performance is still at least the solution of the **NBPS**, which represents the disagreement point of this bargaining problem. Especially, if the NBS exists, it is unique and satisfies the four axioms:
a) **Pareto Efficiency**: NBS produces a Pareto optimal solution, i.e., no operators can improve its communications service quality without compromising the others'.
b) **Symmetry**: NBS provides equal gains from cooperation when the feasible region is symmetric, where the feasible region is agnostic of the player identities. As a result, the solution will be the same even if the operators utility axis are swapped.
c) **Independence of Affine Transformations**: NBS should be agnostic of any affine transformations of operator utilities. Therefore, consider an example of three operators as in Figure 1, if the NBS is given by (ϕ_1^{NB}, ϕ_2^{NB}, ϕ_3^{NB}) for some utilities (ϕ_1, ϕ_2, ϕ_3), and ϕ_1 is transformed to $a_1\phi_1 + b_1$, then the solution changes to ($a_1\phi_1^{\text{NB}} + b_1$, ϕ_2^{NB}, ϕ_3^{NB}).
d) **Independence of Irrelevant Alternatives**: The addition of irrelevant alternatives will not change the NBS. That is, for feasible regions Θ and Θ', if (ϕ_1^{NB}, ϕ_2^{NB}, ϕ_3^{NB}) $\in$ solution(Θ), $\Theta' \subset \Theta$, and (ϕ_1^{NB}, ϕ_2^{NB}, ϕ_3^{NB}) $\in \Theta'$ then (ϕ_1^{NB}, ϕ_2^{NB}, ϕ_3^{NB}) $\in$ solution(Θ').

The NBS of the backup power fair sharing problem can be achieved by solving the following problem

Backup Power Fair Sharing (BPFS):

$$\max_{\mathbf{p}} \quad \prod_{i\in\mathcal{G}} \left[\hat{\phi}_i - \phi(\rho_i)\right]^{\omega_i} \tag{6}$$

$$\text{s.t.} \quad \sum_{i\in\mathcal{G}} \psi_{ij}(\rho_{ij}) \le \sum_{i\in\mathcal{G}} B_{ij}, \qquad \forall j \in \mathcal{H}_i, \tag{7}$$

$$\phi(\rho_i) \le \hat{\phi}_i, \qquad \forall i \in \mathcal{G}, \tag{8}$$

$$\rho_{ij} \in \mathcal{F}_i, \qquad \forall i, j. \tag{9}$$

The $\mathbf{BPFS}$ problem maximizes the product of operators' gains in delay performance over the disagreement point, $\hat{\phi}_i$, which is a constant in the $\mathbf{BPFS}$ problem. The different power coefficients ω_i represent the operator heterogeneity in the fairness design. Using power coefficients allows the objective to become more biased towards the players having higher bargaining power. The inequality constraint (7) will not allow the total power consumption of the BSs of all the operators greater than their total backup power capacity at every site. The constraint (8) enforces the benefit of cooperation for sharing over no sharing. The constraint (9) guarantees the feasibility of BS loads. The optimal user association distribution and BS loads of this problem guarantee a better performance than or equal to the disagreement point. For that reason, disagreement points can be considered as the substitute solutions when all operators cannot achieve better flow-level performance.

B. Decentralized Solution Method

Since solving the $\mathbf{BPFS}$ problem by a centralized controller requires traffic load information of all operators, we derive a decentralized algorithm based on the Jacobi-Proximal Alternating Direction Method of Multipliers (JP-ADMM) approach [21] to protect each operator's private traffic information.

The $\mathbf{BPFS}$ problem is transformed into an equivalent problem as follows

$\mathbf{BPFS}'$:

$$\max_{\mathbf{p}} \quad \sum_{i} \omega_i \ln\left[\hat{\phi}_i - \phi(\rho_i)\right] \tag{10}$$

$$\text{s.t.} \quad \sum_{i\in\mathcal{G}} \psi_{ij}(\rho_{ij}) + b_j = \tilde{B}_j, \qquad \forall j \in \mathcal{H}_i, \tag{11}$$

$$b_j \ge 0, \rho_{ij} \in \mathcal{F}_i, \qquad \forall i, j. \tag{12}$$

Note that the solution of the $\mathbf{BPFS}'$ problem always satisfies the constraint (8). In addition, we introduce slack variables b_j to transform the inequality sharing backup power constraint (7) into the equality constraint (11), where the aggregate power capacity at each site j is defined as $\tilde{B}_j = \sum_{i\in\mathcal{G}} B_{ij}$. As a result, the optimal solution of $\mathbf{BPFS}'$ is also the optimal solution of $\mathbf{BPFS}$ problem.

Lemma 1. *The* $\mathbf{BPFS}'$ *problem is a concave problem due to the objective is a concave function and feasible set is a convex set.*

Proof: The function $g(\rho_i) = \hat{\phi}_i - \phi(\rho_i)$ is a concave function due to $\phi(\rho_i)$ is a convex function. In addition, the composition of a log-concave function, which is nondecreasing in $\mathbb{R}^+$ with the concave function $g(\rho_i)$ is a concave function [22]. Accordingly, the objective function of problem (10), which is the summation of concave functions is also a concave function. Furthermore, the feasible loads set $\mathcal{F}_i$ was proved to be a convex set according to [17]. Since the feasible set of problem (10) is an intersection of convex sets, it is also a convex set. ■

Algorithm 1

1: **Initialization:** Initialize $k = 0, \epsilon$, $b^{(1)}$, and $\lambda^{(1)}$;
2: Each operator i computes $\hat{\phi}_i$ from $\mathbf{NBPS}$ problem (5);
3: **repeat**
4: $\quad k \leftarrow k + 1$
5: $\quad$ Each operator i receives $\lambda^{(k)}, b^{(k)}$;
6: $\quad$ Compute $\psi_{ij}^{(k+1)}$ from subproblem (14);
7: $\quad$ Send $\psi_{ij}^{(k+1)}$ to the coordinator of BSs;
8: $\quad$ Each site j updates the slack variable according to (16) and the dual variable according to (19);
9: **until** $\|\lambda^{(k+1)} - \lambda^{(k)}\| \le \epsilon$.
10: Operator i uses $\mathbf{p}_i^{(k+1)}(x)$ for user association.

There are several decentralized methods can split the resource sharing $\mathbf{BPFS}'$ problem into the individual subproblem of operators that can keeps the operator privacy, i.e., dual decomposition [22], and ADMM [23]. In this work, we adopt one of the state of the art ADMM variants, which is Jacobi-Proximal ADMM [21]. This approach is proposed to cope with faster convergence than dual decomposition method while providing a parallelization structure for subproblems update in the conventional Gauss-Seidel ADMM method [21]. Although Gauss-Seidel ADMM requires fewer iterations for convergence than JP-ADMM as shown in the simulation results of [24], Gauss-Seidel ADMM needs to perform alternatively its subproblems update, thus weakening the scalability in practice. Different from the original Jacobi-ADMM technique, JP-ADMM includes additional proximal terms in subproblems and a new parameter, $\alpha > 0$, for dual variable updates as shown later in update steps.

The augmented Lagrangian is derived for the $\mathbf{BPFS}'$ problem as follows

$$\mathcal{L}_{\mathcal{A}} = -\sum_{i\in\mathcal{G}} \omega_i \ln\left[\hat{\phi}_i - \phi(\rho_i)\right] - \sum_{j\in\mathcal{H}_i} \lambda_j \Big(\sum_{i\in\mathcal{G}} \psi_{ij} + b_j - \tilde{B}_j\Big) + \frac{\rho}{2} \sum_{j\in\mathcal{H}_i} \Big(\sum_{i\in\mathcal{G}} \psi_{ij} + b_j - \tilde{B}_j\Big)^2. \tag{13}$$

The summary of the JP-ADMM based algorithm for backup power sharing is presented in Algorithm 1. At each iteration k of Algorithm 1, operator i receives the dual variables, slack variable and estimated BSs power usage from the previous iteration then individually solves its subproblem to obtain a

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solution for the user association vector $\mathbf{p}_i^{(k+1)}(x)$ as follows

$$\begin{aligned} \min_{\mathbf{p}_i} \quad & -\omega_i \ln\big[\hat{\phi}_i - \phi(\rho_i)\big] + \frac{\tau_i}{2}\sum_{j\in\mathcal{H}_i}(\psi_{ij} - \psi_{ij}^{(k)})^2 \\ & + \frac{\rho}{2}\sum_{j\in\mathcal{H}_i}\Big(\psi_{ij} + \sum_{n\neq i}\psi_{nj}^{(k)} + b_j^{(k)} - \tilde{B}_j - \frac{\lambda_j^{(k)}}{\rho}\Big)^2 \qquad (14) \\ \text{s.t.} \quad & \rho_{ij} \in \mathcal{F}_i, \qquad \forall j \in \mathcal{H}_i. \end{aligned}$$

The square differences between the power consumption variables and the previous iteration solutions are known as proximal terms.

Sites updates: After solving the subproblem (14), each operator sends its estimated power consumption $\psi_{ij}^{(k+1)}$ given the user association solutions at the current iteration to the coordinators at co-located sites. Then, the coordinator at each site j updates the slack variables $b_j^{(k+1)}$ as follows:

$$\min_{b_j\geq 0} \ \frac{\rho}{2}\Big(\sum_{i\in\mathcal{G}}\psi_{ij}^{(k+1)} + b_j - \tilde{B}_j - \frac{\lambda_j^{(k)}}{\rho}\Big)^2 + \frac{\tau_j}{2}(b_j - b_j^{(k)})^2. \qquad (15)$$

This slack variable update also needs an additional proximal term due to its appearance in consensus sharing constraint.

Lemma 2. *The optimal solution of the problem (15) is achieved by using Karush-Kuhn-Tucker (KKT) condition [25] as follows*

$$b_j^{(k+1)} = \Big[\frac{\rho\big(\tilde{B}_j - \sum_{i\in\mathcal{G}}\psi_{ij}^{(k+1)}\big) + \lambda_j^{(k)} + \tau_j b_j^{(k)}}{\rho + \tau_j}\Big]^+. \qquad (16)$$

Proof: We derive the Lagrangian of problem (15) with Lagrangian multiplier $\mu \geq 0$ as follows

$$\begin{aligned} \mathcal{L}(b_j,\mu_j) = {} & \frac{\rho}{2}\Big(\sum_{i\in\mathcal{G}}\psi_{ij}^{(k+1)} + b_j - \tilde{B}_j - \frac{\lambda_j^{(k)}}{\rho}\Big)^2 \\ & + \frac{\tau_j}{2}(b_j - b_j^{(k)})^2 - \mu_j b_j. \end{aligned}$$

Using KKT condition, we first have the following criterion

$$\begin{aligned} & \frac{\partial\mathcal{L}}{\partial b_j} = 0 \\ & \Leftrightarrow \rho\Big(\sum_{i\in\mathcal{G}}\psi_{ij}^{(k+1)} + b_j^* - \tilde{B}_j - \frac{\lambda_j^{(k)}}{\rho}\Big) + \tau_j(b_j^* - b_j^{(k)}) = \mu_j \\ & \Leftrightarrow b_j^* = \Big(\frac{\rho\big(\tilde{B}_j - \sum_{i\in\mathcal{G}}\psi_{ij}^{(k+1)}\big) + \lambda_j^{(k)} + \tau_j b_j^{(k)} - \mu_j}{\rho + \tau_j}\Big). \qquad (17) \end{aligned}$$

From complementary slackness criterion, we also have

$$\mu_j^* b_j^* = 0, \mu_j^* \geq 0, b_j^* \geq 0. \qquad (18)$$

Therefore, from (17) and (18), we get the closed-form of the slack variable as (16). ∎

Finally, the coordinator updates dual variables as follows

$$\lambda_j^{(k+1)} = \lambda_j^{(k)} - \alpha\rho\Big(\sum_{i\in\mathcal{G}}\psi_{ij}^{(k+1)} + b_j^{(k+1)} - \tilde{B}_j\Big). \qquad (19)$$

The algorithm keeps iteratively updating variables until the dual variables differences below the predefined threshold.

Under the mild conditions, i.e., the splittable objective functions are closed proper convex and the existence of a saddle point satisfying KKT condition, the sufficient condition of JP-ADMM for the global convergence to the saddle point according to Theorem 2.1 in [21] can be guaranteed by choosing parameters such that

$$\tau > \rho\Big(\frac{|\mathcal{G}|}{2-\alpha} - 1\Big), \text{ and } 0 < \alpha < 2,$$

where $|\mathcal{G}|$ is the total number of operators. Moreover, with additional running conditions, JP-ADMM achieves $o(1/k)$ convergence rate from Theorem 2.2 in [21], where k denotes the number of iterations. This convergence rate defines a quantify for the complexity of the decentralized optimization approach [21]. Specifically, $\|x^k - x^{k+1}\|^2_{M_x} = o(1/k)$ and $\|\lambda^k - \lambda^{k+1}\|^2 = o(1/k)$ where x^k is the primal solution, λ^k is the dual solution at the iteration k and M_x is defined in [21]. Conceivably, the gaps between solutions become gradually smaller and the primal, dual solutions converge toward the optimal ones throughout iterative updates in JP-ADMM algorithm.

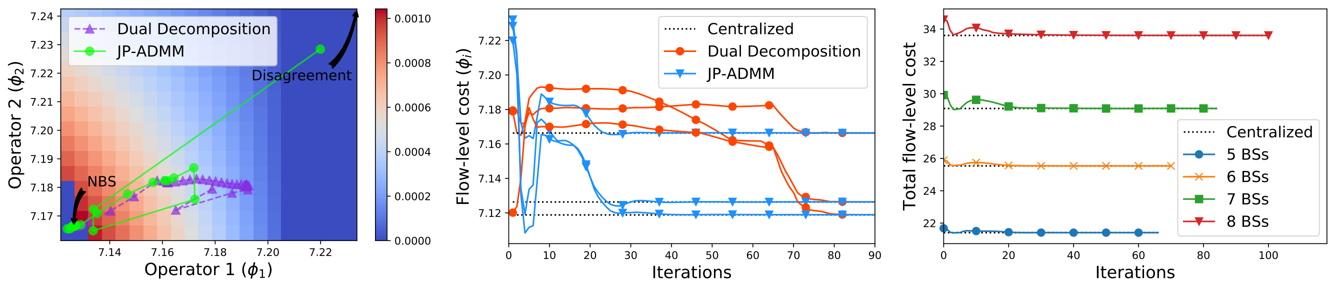
The decentralized algorithm only requires sharing the dual variables, slack variables, and estimated BSs power usage with other operators while keeping traffic flows and user association information of each operator private.

C. Case Studies

In this section, we will demonstrate the convergence of the proposed decentralized algorithm, and compare the reduced average delay and power consumption of the sharing backup power with that of the no sharing case.

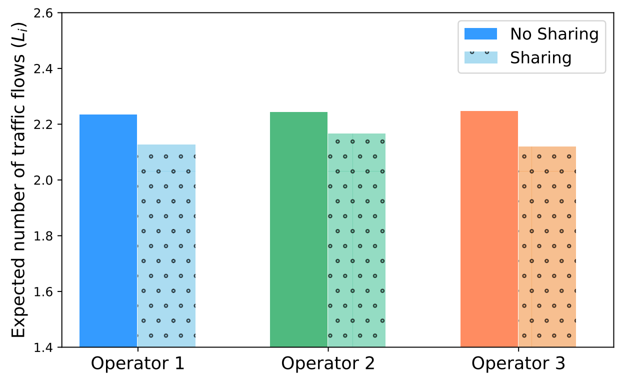
1) Simulation Settings: For an example scenario, we consider three operators which are co-located at five sites and share their infrastructure as in Figure 1. In this scenario, user traffic flows can be associated with all BSs and affect to the BS utilization. These sites are located randomly in a $1\times 1\,km^2$ region, which is divided into 100 unit squares. The location x of data requests is determined at the bottom left corner of each unit area. According to the communication model of urban macro cells with simulation parameters in the WiMAX evaluation methodology document [26], we use the COST 231 path loss model with BS height $32\,m$ and MT height $1.5\,m$. In the simulation, we consider no inter-operator interference and static Gaussian-like noise inter-cell interference with log-normal shadow fading with standard deviation $8\,dB$ and the maximum BSs transmission power is $40\,W$. The backup power capacity at the multi-operator sites is downsized to $388\,W$ per operator while the maximum BS operational power is $865\,W$ [18].

We assume that each data request has the size that is log normally distributed with mean $1/\mu(x) = 1$. As an example of the heterogeneity of service demands, the traffic loads of operators in the considered region are generated by decreasing arrival rate from the top left and bottom right corner to the secondary diagonal for operator 1 while in the reverse direction for operator 3. On the other hand, operator 2 has the high

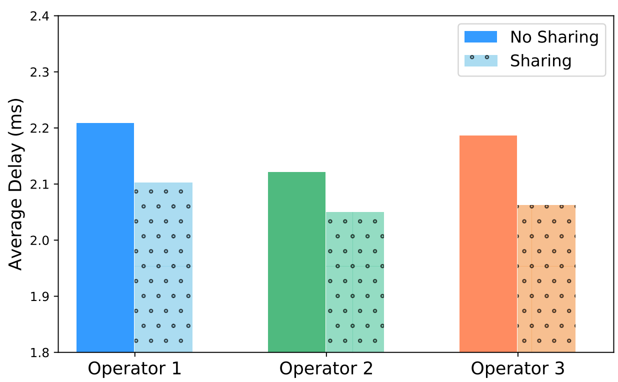


(a) Convergence trajectory of algorithms (b) Convergence rate of algorithms (c) Effects of varying the number of BSs

Fig. 2: Operator's cost convergence.



(a) Expected number of traffic flows of each operator.



(b) Average flow delay of each operator.

Fig. 3: Performance improvement of via sharing.

arrival rate near the central BS (i.e. BS3) while low arrival rate near the other BSs. Finally, in the simulation results, we consider operators are homogeneous in fairness objective, i.e. $\omega_i = 1$.

2) Simulation Results: Figure 2 shows the convergence of flow-level cost of the decentralized algorithms compared with the optimal solution of the centralized algorithm (i.e., IpOpt solver [27]). Using the same setting of traffic loads and initial parameters, our JP-ADMM based decentralized algorithm produces faster convergence to the optimal solution than dual decomposition method as shown in Figure 2b. Note that the estimated power usage of each iteration solution can be over the limited capacity of backup power. In this case, the partial augmented Lagrangian has high penalty values. As a result, the JP-ADMM algorithm passed through low cost values, especially, at the iteration 4, 5, and 6. We also observe that different initial parameters strongly affect to dual decomposition convergence while it is more consistent in case of JP-ADMM. Specifically, in Figure 2a, we can observe different convergence trajectories between two decentralized algorithms. In this simulation result, we fix the flow-level cost of operator 3, which belongs the NBS of **BPFS** problem and vary flow-level cost of the remaining two operators. Accordingly, the color region represents the flow-level cost region of operator 1 and 2, which can vary from the NBS point to the disagreement point and the color values present the product of operators' gains according to the objective (6) of **BPFS** problem. This product increases along with the increment of both operators' cost and achieves the maximum value at NBS, i.e., a Pareto solution. As we expected, both algorithms converge to the NBS point by solving **BPFS**$'$ problem. Although at the beginning, JP-ADMM solution is far from NBS, it moves quickly to the NBS after several iterations and converges faster than dual decomposition method. Furthermore, Figure 2c indicates the JP-ADMM algorithm requires more number of iterations when we increase the number of BSs in the region and keep the same stopping condition threshold for all scenarios. The more required number of iterations leads to the more running time (i.e., computational time of the subproblem, variables exchange, and site update time) for the decentralized algorithm to converge. In this result, after 40 iterations, the total cost improvement is negligible for all of the cases.

As a result of NBS, compared with no sharing, the sharing approach increases the expected number of flows, averaged over all operators, up to 4.8% and reduces average delay by 4.6% in Figure 3a, and 3b, respectively. The derived results based on queueing analysis show that the higher expected number of flows, the faster queue can process, thus the lower average delay according to Little Law as in the equation (2). In addition to the improvement of the average delay in Figure 3b, the backup power efficiency, averaged over all operators, can be increased by 3.6% as shown in Figure 4.

Figure 5 illustrates the user association distribution of the central BS3 and the coverage of other BSs belong to operator 1. In this result, we examine the coverage of user association distribution according to heavy traffic load areas near BS2 and

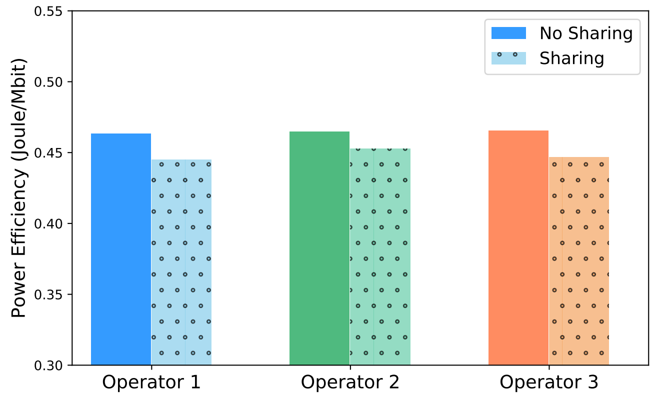
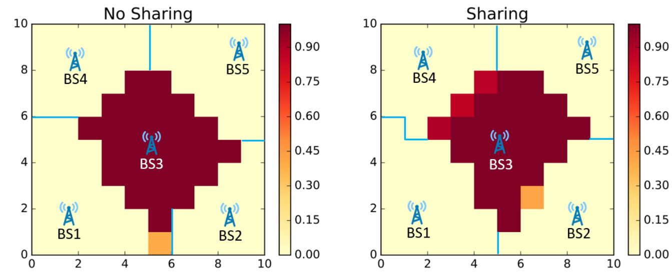


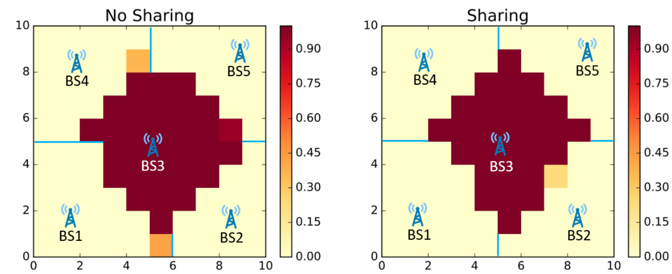
Fig. 4: Power efficiency of operators.

BS4. The red areas show the user association distribution of BS3 while the yellow areas are the coverage of the remaining BSs. The orange squares illustrate the locations where traffic flows can be probabilistically associated with multiple BSs. Due to the path loss effect, user flows try to associate with the closer BSs to receive higher transmission rates. In addition, the BSs load balancing and backup power constraint design forces some user flows to associate with the farther low-utilized BSs. As a result, the BSs near heavy traffic load density areas will have small coverages. Specifically, when the backup power capacity of BS3 is set to $388\,W$, the BS2 and BS4 have smaller coverage than other BSs, as illustrated in Figure 5a. With power sharing, there are more MTs being able to associate with their closest BS. Hence, BS2 and BS4 coverage become larger with the sharing backup power approach. Figure 5b illustrates the user association distribution of the BS3 and the coverage of the other BSs of operator 1 when backup power capacity is increased to $500\,W$. The higher power capacity allows more MTs to associate with BS2 and BS4 and the coverage of these BSs become larger. The coverage of BS1 and BS5 that are near the low traffic areas are reduced for both sharing and no sharing cases.

We next investigate the effect of increasing backup power capacity. In the $\mathbf{NBPS}$ and $\mathbf{BPFS}$ problems, the flow-level performance is determined by the user association solution, which depends on the limitation of BS backup power. Figure 6 illustrates that the total flow-level cost of operators decreases along with the increasing of the backup power capacity for both no sharing and sharing backup power case. The lower backup power capacity produces the higher improvement in flow-level performance by sharing backup power compared to no sharing scheme. However, when the backup power capacity becomes greater than $440\,W$, sharing and no sharing approaches have almost similar performance since all BSs become low utilized. Accordingly, the sharing approach does not show the benefit for low-utilized scenarios, such as excessive backup power capacity provision or low traffic loads in night hours. However, during emergency situations, user traffic loads are generally high because many people may be panic and try to search necessary information. Therefore, the study on fair sharing becomes the vital issue in highly-utilized scenarios.



(a) Backup power capacity of BS3 is 388 W.



(b) Backup power capacity of BS3 is 500 W.

Fig. 5: User association distribution of the BS3 of operator 1 over the region $\mathcal{L}$.

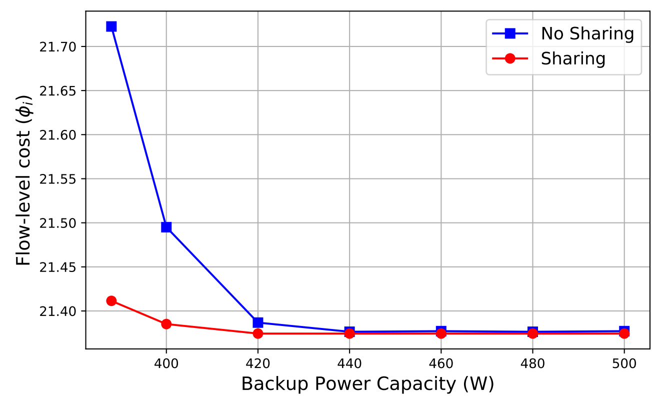


Fig. 6: Total flow-level cost when increasing backup power capacity of BSs.

IV. Backup Energy Sharing and Planning

For disaster scenarios, which happen with significant damages and take a long time to recover the power grid system, BSs can only use the backup power to prolong the communication services over a limited period because the energy of power supply gets constantly depleted over usage time. Therefore, service availability of the BSs during the prolonged power outages is constrained by available energy. In this context, the user association problem should be optimized while taking future service availability into account. Planning ahead the remaining backup energy for future traffic loads is an important goal to improve the performance in this scenario.

A. Problem Formulation

In this subsection, suppose that there is a utility power outage affecting the wireless operators' BSs in the service area. We will present two energy planning problems for this emergency case. While the first problem relates to individual power planning without backup power sharing, the second one examines both backup power fair sharing and energy planning problems.

1) Individual power planning (IPP): When the power outage occurs, each operator needs to plan its energy usage using the predicted traffic flows under energy capacity constraints. In this scenario, a finite time horizon is divided into T sample time periods. Without backup energy sharing, at the current time $1 \leq t \leq T$, the user association plan of each operator i for the remaining time periods is solved by the following problem

$\mathbf{IPP}$:

$$\min_{\mathbf{p}_i(t)} \quad \sum_{\tau=t}^{T} \phi(\rho_i(\tau)) \tag{20}$$

$$\text{s.t.} \quad \sum_{\tau=t}^{T} \psi_{ij}(\rho_{ij}(\tau)) \times \Delta_t \leq E_{ij}(t), \qquad \forall j \in \mathcal{H}_i, \tag{21}$$

$$\rho_{ij}(\tau) \in \mathcal{F}_i, \qquad \forall j \in \mathcal{H}_i, \quad \tau = t \dots T. \tag{22}$$

Intuitively, the $\mathbf{IPP}$ problem can be seen as the aggregate of $\mathbf{NBPS}$ problems over the remaining time of the control horizon with the energy limitation constraints (21). The constraint (22) guarantees that the planning solution produces feasible BS loads. Even though there are several approaches to solve the $\mathbf{IPP}$ problem, we choose Model Predictive Control (MPC), a heuristic approach, for this problem. We will explain the MPC method in details in the next subsection. With MPC, only the solution for the current time of user association plan is applied and the remaining energy is updated for the next time period as follows:

$$E_{ij}(t+1) = E_{ij}(t) - \psi_{ij}(\rho_{ij}(t)) \times \Delta_t, \qquad \forall i, j. \tag{23}$$

The remaining backup energy is decreased by the amount of a product of usage power at the certain time with the length of time period Δ_t.

2) Backup Energy Fair Sharing and Planning (BESP): To design a cooperative strategy for backup energy fair sharing and planning, we formulate the following problem for each time period t of the finite horizon T:

BESP:

$$\max_{\mathbf{p}(t)} \quad \prod_{i \in \mathcal{G}} \Big(\sum_{\tau=t}^{T} \big(\hat{\phi}_i(\tau) - \phi(\rho_i(\tau)) \big) \Big)^{\omega_i} \tag{24}$$

$$\text{s.t.} \quad \sum_{\tau=t}^{T} \sum_{i \in \mathcal{G}} \psi_{ij}(\rho_{ij}(\tau)) \times \Delta_t \leq \sum_{i \in \mathcal{G}} E_{ij}(t), \ \forall j \in \mathcal{H}_i, \tag{25}$$

$$\sum_{\tau=t}^{T} \phi(\rho_i(\tau)) \leq \sum_{\tau=t}^{T} \hat{\phi}_i(\tau), \qquad \forall i \in \mathcal{G}, \tag{26}$$

$$\rho_{ij}(\tau) \in \mathcal{F}_i, \qquad \forall i, j; \tau = t \dots T. \tag{27}$$

At a given current time period t, $\mathbf{BESP}$ problem maximizes the product of the incremental aggregate average delay performance of all operators over the remaining time. This improvement will be guaranteed by the constraint (26). When all operators participate in the designed cooperative game, the individual resource constraints become shared backup energy constraints (25).

We combine both Nash Bargaining and MPC approaches for our algorithm design. Similar to the previous section, NBS can guarantee the Pareto-efficiency and proportional fairness to enable the operators' willingness of cooperation. In fact, there are many solution approaches for solving $\mathbf{BESP}$, e.g., dynamic programming, MPC. In this work, we choose MPC with the simpler complexity and more practical implementation than dynamic programming [28], which incurs the "curse of dimensionality", where the computational complexity of the dynamic programming algorithm increases exponentially with the dimensionality of the state. MPC is designed for predicting and solving an optimization problem over a finite horizon time. Even though MPC produces a suboptimal control, in practice, it has been widely applied in several control problems [29], e.g., power control [30]–[32]. Using MPC [28], [33], the operators can make decisions which account for future responses to spread out the energy usage over the control horizon under the limitation of BSs energy. From the current BS state observation, an optimal user association plan is computed for the future remaining time horizon with the predicted future remaining backup energy states as follows

$$\sum_{i \in \mathcal{G}} E_{ij}(t+1) = \sum_{i \in \mathcal{G}} E_{ij}(t) - \sum_{i \in \mathcal{G}} \psi_{ij}(\rho_{ij}(t)) \times \Delta_t. \tag{28}$$

Compared with the energy update (23) in $\mathbf{IPP}$, the remaining backup energy update (28) is the summation of individual available energy, which is decreased by the amount of energy usage in the current time period. Only the optimal user association at the certain time slot t, which is the first step of the optimal control horizon $\{t \dots T\}$, is applied. The available backup energy is reduced by the amount of energy usage in this time period according to (28). Then, the traffic load density is sampled for the next time period and the process will be repeated for the remaining time of the control horizon.

Then, using logarithm transformation, we obtain the equivalent problem of $\mathbf{BESP}$ as follows

$$\max_{\mathbf{p}(t)} \quad \sum_{i \in \mathcal{G}} \omega_i \ln \Big(\sum_{\tau=t}^{T} \hat{\phi}_i(\tau) - \phi(\rho_i(\tau)) \Big) \tag{29}$$

$$\text{s.t.} \quad \text{Constraints (25), (27).}$$

As a similar approach to the previous section, we introduce a slack variable to transform the inequality constraint (25) to an equality constraint and simplify the notations as in the following $\mathbf{BESP'}$ problem:

$$\max_{\mathbf{p}(t)} \quad \sum_{i \in \mathcal{G}} \omega_i \ln \Big(\sum_{\tau=t}^{T} \hat{\phi}_i(\tau) - \phi(\rho_i(\tau)) \Big) \tag{30}$$

$$\text{s.t.} \quad \sum_{i \in \mathcal{G}} \zeta_{ij}(t) + e_j = \tilde{E}_j(t), \quad \forall j \in \mathcal{H}_i, \tag{31}$$

$$e_j \geq 0, \rho_{ij}(\tau) \in \mathcal{F}_i, \qquad \forall i, j; \tau = t \dots T. \tag{32}$$

Algorithm 2

Initialization: Initialize ϵ and each operator i computes the plan $\hat{\phi}_i$ from **IPP** problem;
At each time slot $t = 1 \dots T$
Initialize $k = 0, e^{(1)},$ and $\lambda^{(1)}$;
repeat
  $k \leftarrow k + 1$
  Each operator i receives $\lambda^{(k)}, e^{(k)}$;
  Generate the plan $\rho_i^{(k+1)}(\tau)$ from the operator's subproblem (33) $, \forall \tau = t \dots T$;
  Estimate and send $\zeta_{ij}^{(k+1)}(t)$ to the coordinator of BSs;
  Each site j updates the slack variable according to (35) and the dual variable according to (36);
until $\left\|\lambda^{(k+1)} - \lambda^{(k)}\right\| \le \epsilon$;
Operator i uses $\mathbf{p}_i^{(k+1)}(x)$ for user association at the current time t.
Update the remaining energy of each co-located site j according to (28)

where

$$\tilde{E}_j(t) = \sum_{i\in\mathcal{G}} E_{ij}(t)/\Delta_t, \text{ and } \zeta_{ij}(t) = \sum_{\tau=t}^{T} \psi_{ij}(\rho_{ij}(\tau)).$$

Conceivably, $\tilde{E}_j(t)$ is the average backup power capacity while $\zeta_{ij}(t)$ is the total usage power of each operator i at BS j for the remaining time. We aggregate total usage power into vector form as $\zeta_i(t) = \{\zeta_{ij}(t)\}, \forall j \in \mathcal{H}_i$. The solution of $\mathbf{BESP'}$ problem, if it exists, is also the solution of the **BESP** problem. In addition, to determine the disagreement point $\hat{\phi}_i$ of **BESP** problem, we use the planning solutions of **IPP** problem at the beginning time. Since the problem dimension expands according to the number of time periods, operators, and BSs, solving this high-dimensional problem directly using centralized methods becomes very challenge. Therefore, the distributed approach not only keeps operator information private but also provides an efficient method for dealing with the scalability of the problem.

B. Nash Bargaining with MPC solution to BESP Problem

In this section, the combined NBS with MPC as an integrated solution approach for the $\mathbf{BESP'}$ can be obtained in a decentralized manner using JP-ADMM and MPC update, as shown in Algorithm 2. In addition, Figure 7 illustrates the planning process of three co-located operators to achieve NBS at a certain time t. Each operator will independently solve its user association planning problem based on its user demand, then send the estimated BS power $\zeta_{ij}^{(k+1)}(t)$ to the coordinator at each site. We still need coordinators which can be implemented as light-weight, distributed modules at co-located sites to update slack variables and dual variables. When we do not have a sharing infrastructure owner, any operator can become a representative coordinator. However, the most applicable design is that the shared backup power is an integral element of passive sharing infrastructure and managed by the infrastructure owner. After receiving the

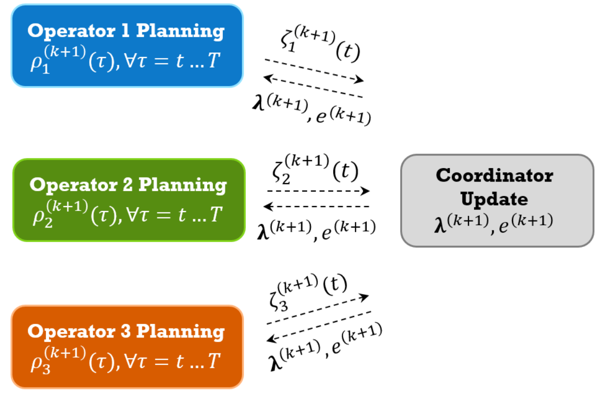


Fig. 7: Planning process of three co-located operators.

new estimated usage power, the coordinator module at site j will update the slack variables $e_j^{(k+1)}$ and dual variables $\lambda_j^{(k+1)}$. This process iteratively performs until the solutions satisfy convergent condition and converge to Nash Bargaining solution of $\mathbf{BESP'}$ problem.

At the certain time t, each operator i iteratively solves its subproblem as follows

$$\begin{aligned} \min_{\mathbf{p}_i(t)} \quad & -\omega_i \ln\Big(\sum_{\tau=t}^{T} \hat{\phi}_i(\tau) - \phi(\rho_i(\tau))\Big) \\ & + \frac{\tau_i}{2} \sum_{j\in\mathcal{H}_i} (\zeta_{ij}(t) - \zeta_{ij}^{(k)}(t))^2 \\ & + \frac{\rho}{2} \sum_{j\in\mathcal{H}_i} \Big(\zeta_{ij}(t) + \sum_{n\neq i} \zeta_{nj}^{(k)}(t) + e_j^{(k)} - \tilde{E}_j(t) - \frac{\lambda_j^{(k)}}{\rho}\Big)^2 \quad (33) \\ \text{s.t.} \quad & \rho_{ij}(\tau) \in \mathcal{F}_i, \qquad \forall j \in \mathcal{H}_i,\ \tau = t \dots T. \end{aligned}$$

Sites updates: After solving the subproblem (33), operators will send their estimated usage power $\zeta_i^{(k+1)}(t)$ given the user association solutions at the current iteration to the coordinators at co-located sites. Then, the coordinator at each site j updates slack variables $e_j^{(k+1)}$ as follows

$$\begin{aligned} \min_{e_j \ge 0} \quad & \frac{\rho}{2}\Big(\sum_{i\in\mathcal{G}} \zeta_{ij}^{(k+1)}(t) + e_j - \tilde{E}_j(t) - \frac{\lambda_j^{(k)}}{\rho}\Big)^2 \\ & + \frac{\tau_j}{2}(e_j - e_j^{(k)})^2 \end{aligned} \quad (34)$$

Similar to the previous section, we can simply derive the solution of e-update via KKT condition as follows:

$$e_j^{(k+1)} = \Big[\frac{\rho\big(\tilde{E}_j(t) - \sum_{i\in\mathcal{G}} \zeta_{ij}^{(k+1)}(t)\big) + \lambda_j^{(k)} + \tau_j e_j^{(k)}}{\rho + \tau_j}\Big]^+. \quad (35)$$

Finally, the coordinator updates dual variables as follows

$$\lambda_j^{(k+1)} = \lambda_j^{(k)} - \alpha\rho\Big(\sum_{i\in\mathcal{G}} \zeta_{ij}^{(k+1)}(t) + e_j^{(k+1)} - \tilde{E}_j(t)\Big). \quad (36)$$

In the decentralized Algorithm 2, before planning, each operator needs to compute its disagreement point $\hat{\phi}_i$ from the **IPP** problem. At a certain period t of the control horizon, after receiving dual variables $\lambda = \{\lambda_j\}$ and slack variables $e = \{e_j\}$ for all $j \in \mathcal{H}_i$ from the coordinator at co-located sites, each operator separately solves the subproblem (33) and sends the estimated total usage power back to the coordinators for later dual variables update in line 9 of Algorithm 2. Intuitively, each operator will update its own planning of backup power usage for the certain time period until converging to the optimal solution of $\mathbf{BESP}$ with given predicted future traffic loads. Accordingly, the decentralized Algorithm 2 only needs to exchange the copy of dual variables instead of private information, such as traffic flows and user association vector of each operator. Using the planning at the certain time, the available energy is updated for the next time period.

C. Case Studies

1) Simulation Settings: Most of the settings follow those in the previous section. In addition, we consider an hour planning with 12 time periods, and each time period is equal to 5 minutes and 16 time periods for 8 hours planning according to the FCC suggestion [34]. Using historical information, operators will predict their future traffic load density from the certain time t for MPC planning. The traffic load density prediction is modeled as $\hat{\gamma}^{(t)}(x) = \gamma^{(t)}(x) + \sigma^{(t)}$, where $\sigma^{(t)}$ is the Gaussian noise with zero mean and higher variance for longer prediction by time. We generate three different traffic load scenarios as in Figure 8a, 8b, and 8c.

2) Simulation Results: We first examine the effect of traffic load density to the flow-level cost of each operator with three traffic profile scenarios in Figure 8. In all of three scenarios, the flow-level cost of each operator follows the similar trend of normalized traffic loads. The MPC planning with no sharing scheme is denoted for the the $\mathbf{IPP}$ solution while MPC planning with sharing backup energy is denoted for the $\mathbf{BESP}$ solution. The figure shows that the operator cost with sharing backup energy is lower than that of no sharing approach for both given exact traffic density and predicted traffic density cases at each location during the considered control horizon in the high traffic load in 1 hour and 8 hours scenarios, as shown in Figure 8a, 8c. Since the prediction errors affect the planning process at some time periods, the sharing algorithm cannot find the bargaining solution and decide to use the user association solution of the no sharing planning. The low traffic scenario does not show the improvement of operator performance as in Figure 8b. However, as in the previous section discussion about the emergency situation, the traffic loads are generally high, thus we focus more on the high traffic load scenario.

We next examine the benefit of sharing and planning energy in Figure 9. In Figure 9a, the average accumulated flow-level cost of all operators in the first traffic scenario is decreased by 0.7% with given exact traffic density, and 0.5% with predicted traffic density. While in Figure 9c, the average accumulated flow-level cost of all operators is reduced up to 0.6% in the 8 hours scenario. From the results, the flow-level performance of MPC sharing strategy with traffic prediction is very close to given exact traffic density information one. These results illustrate the flow-level performance improvement of all operators by using sharing backup energy approach in high traffic scenarios.

Figure 10 shows the remaining energy after cooperative planning period (i.e., the target survival time). Figure 10a shows that the remaining energy of the sharing approach is higher than that of the no sharing scheme up to 31% with given exact traffic density, and 24.8% with predicted traffic density in the first scenario. While Figure 10c shows that the remaining energy of the sharing approach is higher than that of the no sharing scheme up to 24.5% in the 8 hours scenario. Therefore, sharing backup energy not only improves the flow-level performance but also saves more backup energy for all operators than that of the individual planning approach.

V. Conclusions

In this paper, we investigate an under-explored problem of backup power sharing for co-location BSs to improve the network performance and service availability during power outages. The fairness of sharing backup power supply among the operators at multi-operator sites is tackled by using Nash Bargaining solution, which can help to mitigate the flow-level cost and reduce power usage. Then, two decentralized algorithms are proposed for two scenarios corresponding to with and without energy capacity considerations. Simulation results show that the backup power fair sharing guarantees better delay reduction than that of no sharing approach. In addition to the delay reduction, the cooperative fair backup power sharing also decreases the operator's BS power consumption in both scenarios.

References

[1] K. Sudeep, "Blackouts in the power system," *International Journal of Electrical and Electronics Research*, vol. 3, no. 4, pp. 1–7, 2015.

[2] "Blackout Tracker: United States Annual Report 2015," Eaton, Tech. Rep., 2015. [Online]. Available: http://powerquality.eaton.com/blackouttracker/default.asp

[3] "Order on reconsideration, FCC 07-177," Federal Communications Commission, Tech. Rep., 2007. [Online]. Available: http://njslom.org/FCC-07-177A1.pdf

[4] "Aftermath of the Great East Japan Earthquake and KDDI's Efforts," KDDI CSR, Tech. Rep., 2011. [Online]. Available: http://media3.kddi.com/extlib/files/english/corporate/csr/csr_report/2011/pdf/csr_report_2011en_04.pdf

[5] M. Crouch, "Fuel Cell Systems for Base Stations: Deep Dive Study," GSMA, Tech. Rep., 2012. [Online]. Available: http://studylib.net/doc/18456164/fuel-cell-systems-for-base-stations--deep-dive-study

[6] "Mobile Base Stations Report 2016 Ed 3," Statplan Energy Ltd, Tech. Rep., 2016. [Online]. Available: https://www.reportlinker.com/p04230308/Mobile-Base-Stations-Report-Ed-3.html

[7] "Mobile Infrastructure Sharing," GSMA, Tech. Rep., 2012. [Online]. Available: https://www.gsma.com/publicpolicy/wp-content/uploads/2012/09/Mobile-Infrastructure-sharing.pdf

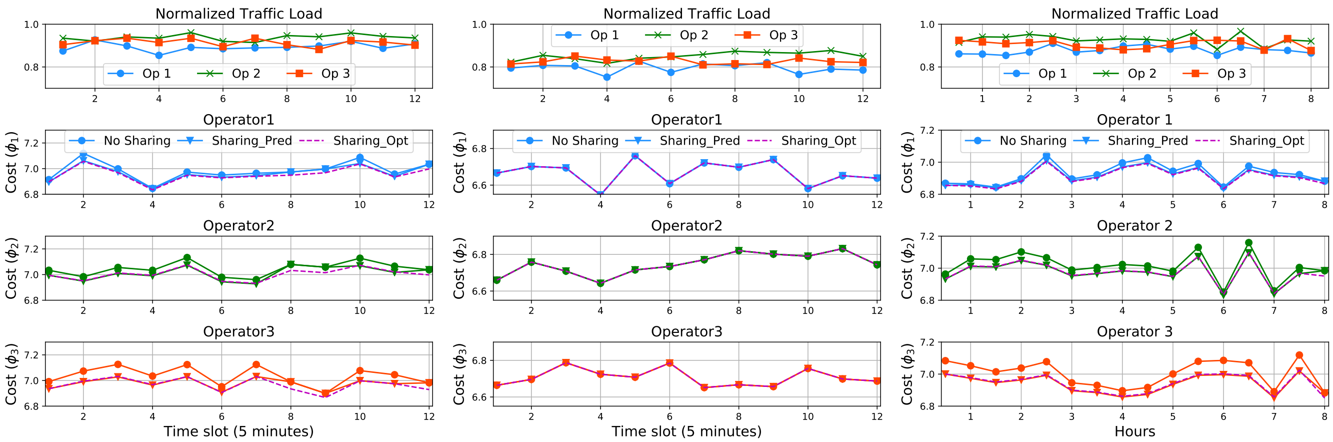
[8] "Infrastructure sharing during emergencies," CSRIC IV, Federal Communications Commission, Tech. Rep., 2014. [Online]. Available: https://transition.fcc.gov/pshs/advisory/csric4/CSRIC_IV_WG-9_Report_061814.pdf

[9] M. J. Osborne and A. Rubinstein, *Bargaining and markets*. Academic press, 1990.

[10] Y. Guo, H. Li, and M. Pan, "Colocation Data Center Demand Response using Nash Bargaining Theory," *IEEE Transactions on Smart Grid*, pp. 1–1, 2017.

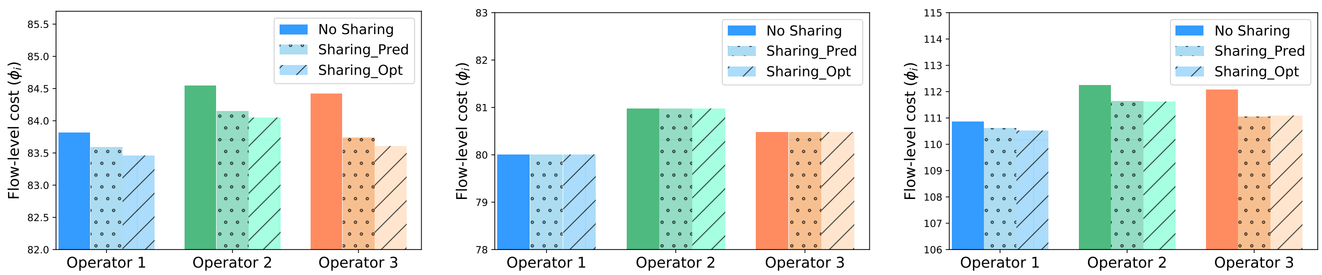
[11] J. Xu and R. Zhang, "Cooperative Energy Trading in CoMP Systems Powered by Smart Grids," *IEEE Transactions on Vehicular Technology*, vol. 65, no. 4, pp. 2142–2153, apr 2016.

[12] Dantong Liu, Yue Chen, Kok Keong Chai, and Tiankui Zhang, "Nash bargaining solution based user association optimization in HetNets," in *2014 IEEE 11th Consumer Communications and Networking Conference (CCNC)*. IEEE, jan 2014, pp. 587–592.



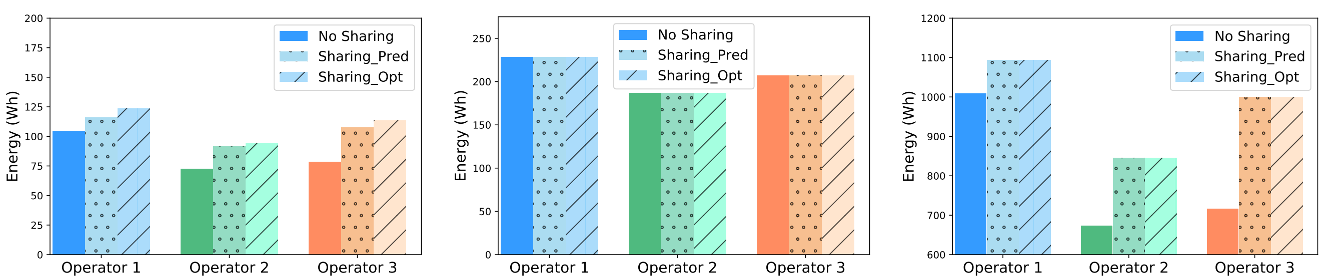
(a) High traffic in 1 hour scenario. (b) Low traffic in 1 hour scenario. (c) High traffic in 8 hours scenario.

Fig. 8: Flow-level cost over time periods.



(a) High traffic in 1 hour scenario. (b) Low traffic in 1 hour scenario. (c) High traffic in 8 hours scenario.

Fig. 9: Performance improvement of via sharing for different traffic scenarios.



(a) High traffic in 1 hour scenario. (b) Low traffic in 1 hour scenario. (c) High traffic in 8 hours scenario.

Fig. 10: Remaining energy of operators after cooperative planning period.

[13] M. J. Farooq, H. Ghazzai, A. Kadri, H. ElSawy, and M.-S. Alouini, "A Hybrid Energy Sharing Framework for Green Cellular Networks," *IEEE Transactions on Communications*, vol. 65, no. 2, pp. 918–934, feb 2017.

[14] M. Oikonomakou, A. Antonopoulos, L. Alonso, and C. Verikoukis, "Fairness in multi-operator energy sharing," in *2017 IEEE International Conference on Communications (ICC)*. IEEE, may 2017, pp. 1–6.

[15] J. Xu, L. Duan, and R. Zhang, "Energy Group Buying With Loading Sharing for Green Cellular Networks," *IEEE Journal on Selected Areas in Communications*, vol. 34, no. 4, pp. 786–799, apr 2016.

[16] A. Bousia, E. Kartsakli, A. Antonopoulos, L. Alonso, and C. Verikoukis, "Game-Theoretic Infrastructure Sharing in Multioperator Cellular Networks," *IEEE Transactions on Vehicular Technology*, vol. 65, no. 5, pp. 3326–3341, may 2016.

[17] H. Kim, G. de Veciana, X. Yang, and M. Venkatachalam, "Distributed alpha-Optimal User Association and Cell Load Balancing in Wireless Networks," *IEEE/ACM Transactions on Networking*, vol. 20, no. 1, pp. 177–190, feb 2012.

[18] K. Son, H. Kim, Y. Yi, and B. Krishnamachari, "Base Station Operation and User Association Mechanisms for Energy-Delay Tradeoffs in Green Cellular Networks," *IEEE Journal on Selected Areas in Communications*, vol. 29, no. 8, pp. 1525–1536, sep 2011.

[19] H. E. E. O. M. Elfadil, M. A. I. Ali, and M. Abas, "Fractional frequency reuse in LTE networks," in *2015 2nd World Symposium on Web Applications and Networking (WSWAN)*. IEEE, mar 2015, pp. 1–6.

[20] J. F. Nash Jr, "The bargaining problem," *Econometrica: Journal of the Econometric Society*, pp. 155–162, 1950.

[21] W. Deng, M.-J. Lai, Z. Peng, and W. Yin, "Parallel Multi-Block ADMM with o(1/k) Convergence," *Journal of Scientific Computing*, vol. 71, no. 2, pp. 712–736, may 2017.

[22] S. Boyd, L. Xiao, A. Mutapcic, and J. Mattingley, "Notes on decomposition methods," *Notes for EE364B, Stanford University*, pp. 1–36, 2007.

[23] S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein, "Distributed Optimization and Statistical Learning via the Alternating Direction Method of Multipliers," *Foundations and Trends® in Machine Learning*, vol. 3,

no. 1, pp. 1–122, 2010.

[24] T. Lin, S. Ma, and S. Zhang, "On the Global Linear Convergence of the ADMM with MultiBlock Variables," *SIAM Journal on Optimization*, vol. 25, no. 3, pp. 1478–1497, jan 2015.

[25] S. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge university press, 2004.

[26] R. Srinivasan, J. Zhuang, L. Jalloul, R. Novak, and J. Park, "IEEE 802.16m Evaluation Methodology Document (EMD) IEEE 802.16m-08/004r5," IEEE 802.16 Broadband Wireless Access Working Group Std., Tech. Rep., 2009. [Online]. Available: http://ieee802.org/16

[27] A. Wächter and L. T. Biegler, "On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming," *Mathematical Programming*, vol. 106, no. 1, pp. 25–57, mar 2006.

[28] D. P. Bertsekas, *Dynamic programming and optimal control*. Athena Scientific, 2005.

[29] S. J. Qin and T. A. Badgwell, "A survey of industrial model predictive control technology," *Control Engineering Practice*, vol. 11, no. 7, pp. 733–764, jul 2003.

[30] J. Rodriguez, M. P. Kazmierkowski, J. R. Espinoza, P. Zanchetta, H. Abu-Rub, H. A. Young, and C. A. Rojas, "State of the Art of Finite Control Set Model Predictive Control in Power Electronics," *IEEE Transactions on Industrial Informatics*, vol. 9, no. 2, pp. 1003–1016, may 2013.

[31] R. T. Bambang, A. S. Rohman, C. J. Dronkers, R. Ortega, A. Sasongko *et al.*, "Energy management of fuel cell/battery/supercapacitor hybrid power sources using model predictive control," *IEEE Transactions on Industrial Informatics*, vol. 10, no. 4, pp. 1992–2002, 2014.

[32] N. Chen, L. Gan, S. H. Low, and A. Wierman, "Distributional analysis for model predictive deferrable load control," in *53rd IEEE Conference on Decision and Control*. IEEE, dec 2014, pp. 6433–6438.

[33] E. F. Camacho and C. B. Alba, *Model predictive control*. Springer Science & Business Media, 2005.

[34] "Report and order, FCC 15-98," Federal Communications Commission, Tech. Rep., 2015.